

Total No. of Questions - 24

Total No. of Printed Pages - 4

Regd.  
No.

A horizontal row of ten empty rectangular boxes, each with a black border, intended for handwriting practice.

### **Part - III**

## MATHEMATICS, Paper - I (A)

## (Algebra, Vector Algebra and Trigonometry)

### (English Version)

*Time : 3 Hours*

**Max. Marks : 75**

**Note :** This question paper consists of three sections A, B and C.

## SECTION A

$$10 \times 2 = 20$$

## I. Very Short Answer Type Questions

- i) Answer all questions.

ii) Each question carries two marks.

1. If  $f : R - \{0\} \rightarrow R$  is defined by  $f(x) = x^3 - \frac{1}{x^3}$ , then show that

$$f(x) + f\left(\frac{1}{x}\right) = 0$$

2. Find the domain of the real valued function  $f(x) = \frac{2x^2 - 5x + 7}{(x-1)(x-2)(x-3)}$ .

3. If  $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$  and  $\det A = 45$ , then find  $x$ .

4. Find the trace of  $\begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$ .
5. Let  $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$  and  $\bar{b} = 3\bar{i} + \bar{j}$ . Find the unit vector in the direction of  $\bar{a} + \bar{b}$ .
6. Find the vector equation of the line passing through the point  $2\bar{i} + 3\bar{j} + \bar{k}$  and parallel to the vector  $4\bar{i} - 2\bar{j} + 3\bar{k}$ .
7. Find the angle between the vectors  $\bar{i} + 2\bar{j} + 3\bar{k}$  and  $3\bar{i} - 2\bar{j} + 2\bar{k}$ .
8. Express  $\frac{(\sqrt{3}\cos 25^\circ + \sin 25^\circ)}{2}$  as a sine of an angle.
9. Express  $\frac{1 - \cos \theta + \sin \theta}{1 + \cos \theta + \sin \theta}$  in terms of  $\tan \frac{\theta}{2}$ .
10. Prove that  $\frac{\tanh x}{\operatorname{sech} x - 1} + \frac{\tanh x}{\operatorname{sech} x + 1} = -2 \operatorname{cosech} x$ , for  $x \neq 0$ .

## II. Short Answer Type Questions.

- i) Attempt any five questions.  
ii) Each question carries four marks.

11. If  $A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$ , then show that  $(AB)' = B' A'$ .
12. Verify whether the triangle formed by the vectors  $3\bar{i} + 5\bar{j} + 2\bar{k}$ ,  $2\bar{i} - 3\bar{j} - 5\bar{k}$ ,  $-5\bar{i} - 2\bar{j} + 3\bar{k}$  is equilateral or not.
13. If  $\theta$  is the angle between vectors  $\bar{i} + \bar{j}$  and  $\bar{j} + \bar{k}$ , then find  $\sin \theta$ .
14. Prove that  $\frac{\cos^3 \theta - \cos 3\theta}{\cos \theta} + \frac{\sin^3 \theta - \sin 3\theta}{\sin \theta} = 3$ .
15. If  $\alpha, \beta$  are the solutions of the equation  $a \cos \theta + b \sin \theta = c$ , where  $a, b, c \in R$  and if  $a^2 + b^2 > 0$  and  $\sin \alpha \neq \sin \beta$ , then show that  $\sin \alpha + \sin \beta = \frac{2bc}{a^2 + b^2}$ .
16. Prove that  $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{s^2}{\Delta}$ .
17. Prove that  $\tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{13} - \tan^{-1} \frac{2}{9} = 0$ .

## III. Long Answer Type Questions.

- i) Attempt any five questions.
- ii) Each question carries **seven** marks.
18. If  $f:A \rightarrow B$ ,  $g:B \rightarrow C$  are any two bijective functions, then prove that  $gof:A \rightarrow C$  is also a bijective function.
19. Using mathematical induction, prove the statement  

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}, \forall n \in N.$$
20.  $x+y+z=1$ ,  $2x+2y+3z=6$ ,  $x+4y+9z=3$ , Solve the system of equations by using matrix inversion method.
21. Show that  $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$
22. Find the shortest distance between the lines  
 $r = 6\bar{i} + 2\bar{j} + 2\bar{k} + \lambda(\bar{i} - 2\bar{j} + 2\bar{k})$  and  $r = -4\bar{i} - \bar{k} + \mu(3\bar{i} - 2\bar{j} - 2\bar{k}).$
23. If  $A+B+C = \frac{\pi}{2}$ , then prove that  
 $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C.$
24. In a  $\Delta ABC$ , prove that  $4(r_1r_2 + r_2r_3 + r_3r_1) = (a+b+c)^2$ .